FINAL MARK

**GIRRAWEEN HIGH SCHOOL**

**MATHEMATICS**

**HALF YEARLY EXAMINATION**

**2016**

**ANSWERS COVER SHEET**

**Name:**

| QUESTION | MARK | H2 | H3 | H4 | H5 | H6 | H7 | H8 | H9 |
|---|---|---|---|---|---|---|---|---|---|
| 1,2 | 2 | √ | | | | | | | √ |
| 3 | 1 | √ | | | √ | | | | √ |
| 4,5 | 2 | √ | | | | | | | √ |
| 6-10 | 5 | √ | | | √ | | | | √ |
| 11 | 15 | √ | | | √ | | | | √ |
| 12 | 15 | √ | | | √ | | | | √ |
| 13ab | 5 | √ | | | √ | | | √ | √ |
| c | 4 | √ | | | √ | | | | √ |
| de | 6 | √ | | | √ | | | √ | √ |
| 14 | 15 | √ | | | √ | | | | √ |
| 15a | 5 | √ | | | √ | | | √ | √ |
| b | 10 | √ | | | √ | √ | | | √ |
| 16 | 15 | √ | | | √ | | | | √ |
| TOTALS | /100 | /100 | | | /96 | /10 | | /16 | /100 |

**HSC Outcomes** **Mathematics**

H1 seeks to apply mathematical techniques to problems in a wide range of practical contexts.

H2 constructs arguments to prove and justify results.

H3 manipulates algebraic expressions involving logarithmic and exponential functions.

H4 expresses practical problems in mathematical terms based on simple given models.

H5 applies appropriate techniques from the study of calculus, geometry, probability, trigonometry and series to solve problems.

H6 uses the derivative to determine the features of the graph of a function.

H7 uses the features of a graph to deduce information about the derivative.

H8 uses techniques of integration to calculate areas and volumes.

H9 communicates using mathematical language, notation, diagrams and graphs.

**GIRRAWEEN HIGH SCHOOL**

**HALF YEARLY EXAMINATION**

# 2016

# MATHEMATICS

*Time allowed - Two hours*
*(Plus 5 minutes' reading time)*

**DIRECTIONS TO CANDIDATES**

- Attempt ALL questions.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Formulae are on laminated sheet provided.
- Board-approved calculators may be used.
- Answers to multiple choice questions should be coloured in on the answer sheet provided.
- Each of questions 11-16 is to be returned on a *separate* piece of paper clearly marked Question 11 , Question 12 , etc. Each piece of paper must show your name. Write END OF SOLUTIONS on your last piece of paper.
- You may ask for extra pieces of paper if you need them.

**PART A: Questions 1 – 10 (Multiple Choice)** ***Write the letter corresponding to the correct answer on your working pages.***

**Question 1**

Correct to 3 significant figures, 5 745 322=

**(A)** 574 **(B)** 575 **(C)** 5 740 000 **(D)** 5 750 000

**Question 2**

The gradient of a line perpendicular to $2x + 3y = 8$ is

**(A)** $\frac{3}{2}$ **(B)** $\frac{2}{3}$ **(C)**- $\frac{3}{2}$ **(D)** $-\frac{2}{3}$

**Question 3**

$$\sum_{n=1}^{7} \mathbf{4n - 1)} =$$

**(A)**27 **(B)** 90 **(C)** 105 **(D)** 23

**Question 4**

The focus of the parabola $(x + 1)^2 = 8(y - 2)$ is

**(A)**$(-1, 2)$ **(B)**$(1, -2)$ **(C)**$(-1, 0)$ **(D)** $(-1, 4)$

**Question 5**

The sum of the roots of the quadratic equation $3x^2 + 2x + 9 = 0$ is

**(A)**-2 **(B)**$-\frac{2}{3}$ **(C)**3 **(D)** Not real

**Question 6**

$$\lim_{x \to 4} \frac{x^2 - 16}{x - 4} =$$

**(A)**0 **(B)**4 **(C)**8 **(D)** Undefined

**Question 7**

A card is drawn from a standard deck (4 cards of each kind, 13 of each suite, 52 cards altogether). The probability that it is red or an Ace is

**(A)**$\frac{1}{13}$ **(B)**$\frac{1}{2}$ **(C)** $\frac{7}{13}$ **(D)** $\frac{15}{26}$

*Examination continues on the following page*

**Question 8**

If $y = x^3 + 3e^{-2x}$, $\frac{dy}{dx} =$

(A) $3x^2 - 6e^{-2x}$ (B) $3x^2 + 6e^{-2x}$ (C) $3x^2 - \frac{3}{2}e^{-2x}$ (D) $3x^2 + \frac{3}{2}e^{-2x}$

**Question 9**

$$\int \frac{1}{(2x+1)^3}.dx =$$

(A) $\frac{1}{8}(2x+1)^4 + C$ (B) $\frac{1}{8(2x+1)^2} + C$ (C) $-\frac{1}{4(2x+1)^2} + C$ (D) $\frac{1}{8(2x+1)^5} + C$

**Question 10** $\int \frac{1}{e^{3x}}.dx =$

(A) $\frac{1}{2e^{2x}} + C$ (B) $\frac{1}{4e^{4x}} + C$ (C $\frac{1}{3e^{3x}} + C$ (D) $-\frac{1}{3e^{3x}} + C$

**PART B: Show all necessary working on your answer pages.**

| **Question 11** *(15 Marks)* | **Marks** |
|---|---|
| **(a)** Find $\frac{dy}{dx}$ if | |
| **(i)** $y = 2\sqrt{x} - \frac{3}{x} + 2e^{3x}$ | **1** |
| **(ii)** $y = 5x^3e^{-2x}$ | **3** |
| **(iii)** $y = \frac{x^2}{x^3-2x+3}$ | **3** |
| **(iv)** $y = e^{\sqrt{x}}$ | **2** |
| **(b)** (i) Find $\int_1^3 2e^{3x} - 6x^2.dx$ | **3** |
| (ii) If $\frac{dy}{dx} = 3e^{2x}$ and $y = e^2$ when x = 1 find the expression for $y$. | **3** |

*Examination continues on the following page*

**Question 12** *(15 Marks)* **Marks**

**(a)** $A(-2,3), B\ (2,5)$ and $C(4,1)$ lie on the number plane. $M$ is the midpoint of $AC$. *(See diagram.)*

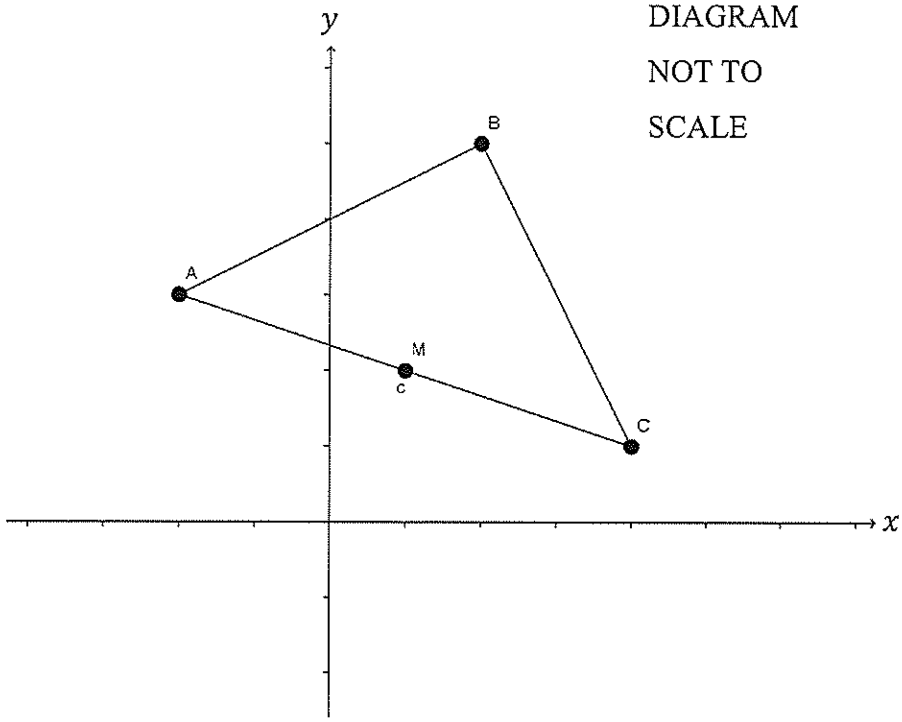



DIAGRAM NOT TO SCALE

| | | |
|---|---|---|
| **(i)** | Show that $AB \perp BC$ | **3** |
| (ii) | Find the coordinates of $M$. | **2** |
| (iii) | Show that $\mathrm{AM} = \mathrm{BM} = \mathrm{CM}$ . | **3** |
| (iv) | The circle which passes through $A, B$ and $C$ has centre $M$. State the equation of this circle. | **1** |

**(b)** A total of 150 tickets are sold in a raffle that has 2 prizes. There are 50 red, 50 green and 50 blue tickets. At the drawing of each prize, winning tickets are NOT replaced before the next draw.

| | | |
|---|---|---|
| (i) | Draw a probability tree showing this information. | **2** |
| (ii) | Find the probability that at least one of the winning tickets is red. | **2** |
| (iii) | Find the probability that the winning tickets are different colours from each other. | **2** |

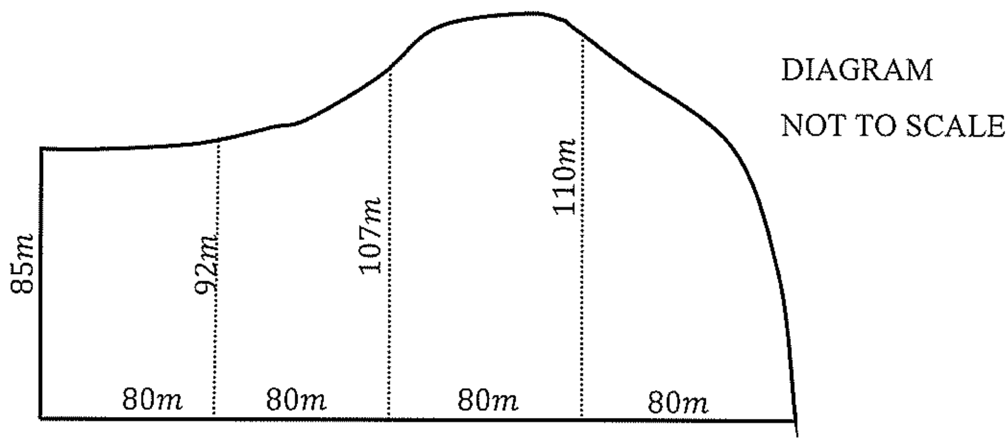
*Examination continues on the following page*

**Question 13** *(15 Marks)* **Marks**

**(a)** Use the trapezoidal with five function values to find an approximation **3**

for $\int_0^1 e^{-x^2}.dx$

**(b)** This irregularly shaped park is bordered by a beach: **2**




Use two applications of Simpson's rule to find the approximate area of the park.

**(c)** (i) If $y = 5xe^x$, find $\frac{dy}{dx}$. **2**

(ii) Hence find $\int 5xe^x.dx$ **2**

**(d)** Find the volume of the solid of revolution formed when $y = e^{3x}$ **2**

is rotated around the $x$ axis between $x = -1$ and $x = 2$.

**(e)** Find the volume of the solid of revolution formed when the area **4**

between the curves $y = 4x, y = \frac{1}{x}$ and the lines $y = 2$ and $y = 3$

is rotated about the $y$ axis.

**Question 14** *(15 Marks)*

**(a)** Ken wins \$500.00 and decides to keep it locked up for a rainy day.

After a while he decides to add more to his stash, so he puts in \$40 one month, then \$55, then \$70 and so on.

(Unfortunately being locked up at home it earns no interest.)

(i) How much money will he put in in the $12^{th}$ month? **1**

(ii) How much money will he have after one year? **2**

(iii) How long will it take Kevin to save \$7000? **3**

*Examination continues on the following page*

**Question 14** *(continued)* **Marks**

**(b)** The number of people attending the Upper Kombuckta West show is increasing by 7% per year. If 3000 people attended the show in the year 2000:

(i) How many people attended the show in 2005? **2**

(ii) How many people attended the show altogether between 2000 and 2007? **2**

**(c)** Find the equation of the tangent to $y = e^{-2x}$ at the point where $x = -1$. **3**

**(d)** If $y = e^{2x} + 3e^{-x}$, show that $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$. **2**

**Question 15** *(15 Marks)*

**(a)** (i) Show by substitution that $y = x^2 - 1$ and $y = 3x + 3$ intersect at the points $(-1,0)$ and $(4,15)$. **2**

(ii) Find the area enclosed by $y = x^2 - 1$ and $y = 3x + 3$ **3**

**(b)** For the function $y = (x + 1)e^x$

(i) Find the coordinates of the $x$ and $y$ intercepts. **2**

(ii) Find any turning points and determine their nature. **3**

(iii) Find any points of inflexion. **3**

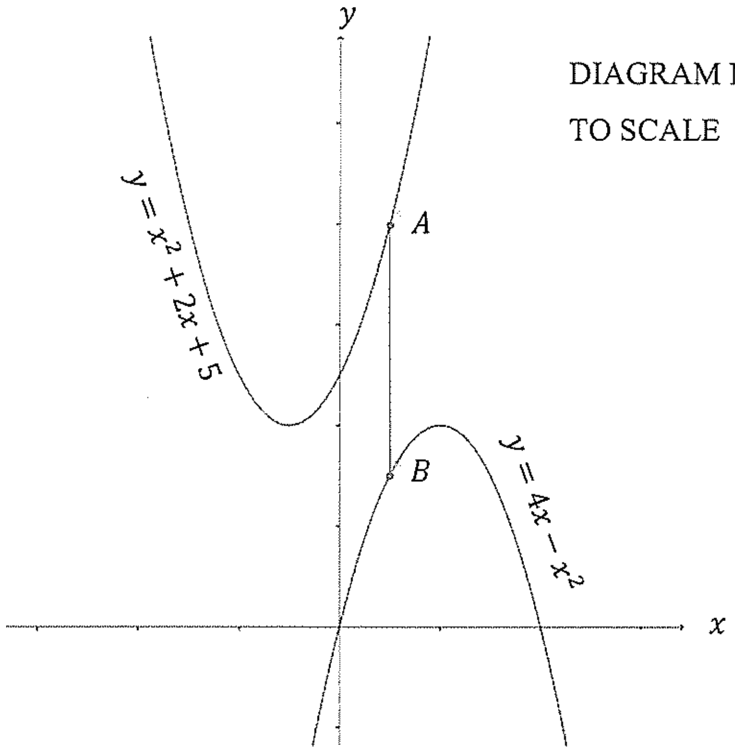
(iv) Sketch the graph of $y = (x + 1)e^x$ showing all intercepts, turning points and points of inflexion. **2**

*Examination continues on the following page*

**Question 16** *(15 Marks)* **Marks**

**(a)** Find the locus of the set of points that are equally distant from the point $(-1, 3)$ and the line $y = -5$. **3**

**(b)** $A$ is a point on the curve $y = x^2 + 2x + 5$. $B$ is a point on the curve $y = 4x - x^2$ with the same $x$ coordinate as $A$. *(see diagram.)*




(i) Using $x$ as the $x$ coordinate of both $A$ and $B$, show that the distance $AB = 2x^2 - 2x + 5$. **2**

(ii) Find the minimum distance $AB$. **4**

**(c)** (i) Write the sum to $n$ terms of the series $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots$. **1**

(ii) For the series $1 + \frac{2}{3} + \frac{3}{9} + \frac{4}{27} + \frac{5}{81} + \cdots . + \frac{n}{3^{n-1}}$

(A) Show that the sum to 5 terms can be written in the form **3**

$$\frac{9}{4}\left[1 - \left(\frac{1}{3}\right)^5\right] - \frac{5}{2} \times \left(\frac{1}{3}\right)^4$$

$$Hint: 1 + \frac{2}{3} + \frac{3}{9} + \cdots .\ can\ be\ written\ as\ 1 + \left(\frac{1}{3} + \frac{1}{3}\right) + \left(\frac{1}{9} + \frac{1}{9} + \frac{1}{9}\right) + \cdots$$

*Examination continues on the following page*

(B) Write an expression for the sum to $n$ terms of **1**

$$1+\frac{2}{3}+\frac{3}{9}+\frac{4}{27}+\frac{5}{81}+\cdots.+\frac{n}{3^{n-1}}$$

(C) Find the limiting sum of the series **1**

$$1+\frac{2}{3}+\frac{3}{9}+\frac{4}{27}+\frac{5}{81}+\cdots.+\frac{n}{3^{n-1}}$$

***Here endeth the examination!!!***

*Remember to write END OF SOLUTIONS on your answer page!!!*

Solutions — Y12 2U Maths Midyear 2016 p-1

(1) D (2) A (3) C (4) D (5) B (6) C (7) C (8) A (9) C (10) D

(11) (a) (i) $y = 2\sqrt{x} - \frac{3}{x} + 2e^{3x}$

$= 2x^{\frac{1}{2}} - 3x^{-1} + 2e^{3x}$

$y' = 2 \times \frac{1}{2} x^{-\frac{1}{2}} - 3 \times -x^{-2} + 6e^{3x}$

$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{x}} + \frac{3}{x^2} + 6e^{3x}$

(ii) $y = 5x^3 e^{-2x}$

$y' = u'v + v'u$

$= 15x^2 e^{-2x} + 5x^3 \times -2e^{-2x}$

$= 15x^2 e^{-2x} - 10x^3 e^{-2x}$

$= 5x^2 e^{-2x}(3 - 2x)$

(iii) $y = \frac{x^2}{x^3 - 2x + 3}$

$y' = \frac{u'v - v'u}{v^2}$

$= \frac{2x(x^3 - 2x + 3) - x^2(3x^2 - 2)}{(x^3 - 2x + 3)^2}$

$= \frac{2x^4 - 4x^2 + 6x - 3x^4 + 2x^2}{(x^3 - 2x + 3)^2}$

$\frac{dy}{dx} = \frac{-x^4 - 2x^2 + 6x}{(x^3 - 2x + 3)^2}$

Q. (11)(a)(iv) $y = e^{\sqrt{x}}$

Either

By if $y = e^{f(x)}$, $y' = f'(x)\, e^{f(x)}$

$$y' = \frac{1}{2\sqrt{x}} \times e^{\sqrt{x}}$$

$$= \frac{e^{\sqrt{x}}}{2\sqrt{x}}.$$

Or

Chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= e^{\sqrt{x}} \times \frac{1}{2\sqrt{x}}$$

$$= \frac{e^{\sqrt{x}}}{2\sqrt{x}}.$$

$u = \sqrt{x}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$y = e^{u}$

$\frac{dy}{du} = e^{u}$

$= e^{\sqrt{x}}$

(b)(i) $\int_1^3 2e^{3x} - 6x^2 \, dx$

$$= \left[\frac{2}{3}e^{3x} - 2x^3\right]_1^3$$

$$= \left[\frac{2}{3} \times e^{3\times 3} - 2\times 3^3\right] - \left[\frac{2}{3}\times e^{3\times 1} - 2\times 1^3\right]$$

$$= \frac{2}{3}e^9 - \frac{2}{3}e^3 - 52$$

$$\doteq 5336.6656 \text{ (4DP)}.$$

(ii) $\frac{dy}{dx} = 3e^{2x}$

$$y = \int 3e^{2x} \, dx$$

$$= \frac{3}{2}e^{2x} + C$$

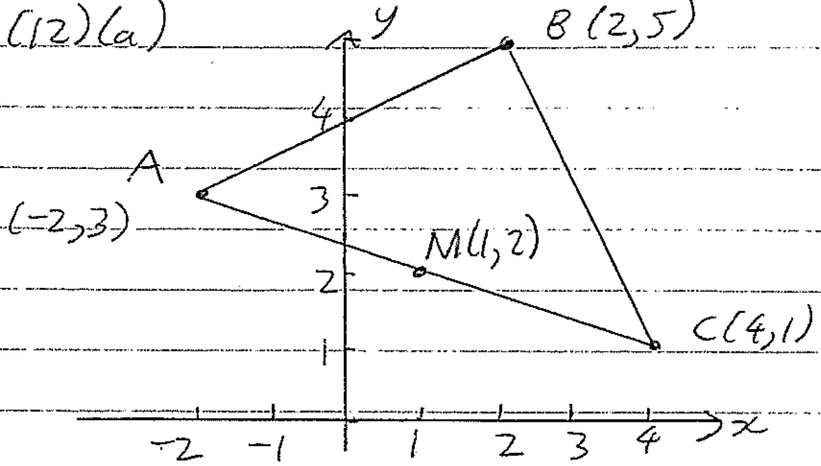
As $y = e^2$ when $x = 1$,

$$e^2 = \frac{3}{2}e^{2\times 1} + C$$

$$-\frac{1}{2}e^2 = C$$

$$\therefore y = \frac{3}{2}e^{2x} - \frac{1}{2}e^2$$

Q.(12)(a)




(i) $m\,AB = \dfrac{y_2-y_1}{x_2-x_1}$ $= \dfrac{5-3}{2+2}$ $= \dfrac{1}{2}$.

$m\,BC = \dfrac{y_2-y_1}{x_2-x_1}$ $= \dfrac{1-5}{4-2}$ $= -2$.

$m\,AB \times m\,BC$
$= \frac{1}{2} \times -2$
$= -1$.

As $m\,AB \times m\,BC = -1$
$AB \perp BC$.

Note*: Alternative below.

(ii) $M = \left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$
$= \left(\dfrac{-2+4}{2}, \dfrac{3+1}{2}\right)$
$= (1, 2)$.

$AC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$= \sqrt{(4+2)^2+(1-3)^2}$
$= \sqrt{40}$

Similarly
$AB = \sqrt{2^2+4^2} = \sqrt{20}$
$BC = \sqrt{2^2+(-4)^2} = \sqrt{20}$
$(AB)^2 + (BC)^2$
$= 20 + 20$
$= 40$
$= (AC)^2$
$\therefore \triangle ABC$ is right-angled at B
(Pythagoras).
$AB \perp BC$.

(iii) $AM = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$= \sqrt{(1+2)^2+(2-3)^2}$
$= \sqrt{10}$

$BM = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$= \sqrt{(2-1)^2+(5-2)^2}$
$= \sqrt{10}$

$CM = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$= \sqrt{(4-1)^2+(1-2)^2}$
$= \sqrt{10}$.
$\therefore AM = BM = CM$.

Alternative for Ext. 1 students
As $\angle ABC = 90^\circ$
B must be at circumference of a circle with AC as diameter [$\angle$ in semicircle]
As M is midpoint AC, M is midpoint of diameter & circle centre
$\therefore AM = BM = CM$ [circle radii].

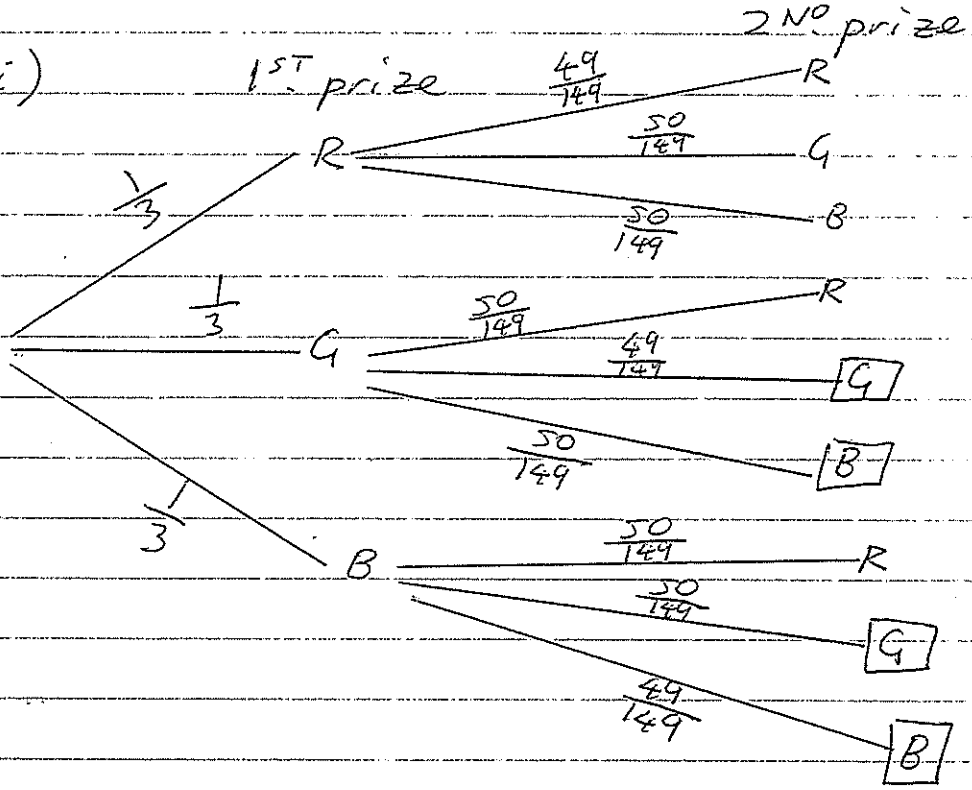
PTO →

Q. (12)(a) (continued):

(iv) Circle centre M, $r=\sqrt{10}$

$(x-1)^2+(y-2)^2=10.$

(b) (i)




(ii) P (at least 1 red)

$= 1 - P(\text{Blue \& Green}) \quad \square$

$= 1 - \left(\frac{1}{3}\times\frac{49}{149}+\frac{1}{3}\times\frac{50}{149}+\frac{1}{3}\times\frac{50}{149}+\frac{1}{3}\times\frac{49}{149}\right)$

$= 1-\frac{66}{149}$

$= \frac{83}{149}$

or $= RR + RG + RB + GR + BR$

$= \frac{1}{3}\times\frac{49}{149}+2\times\frac{1}{3}\times\frac{50}{149}+2\times\frac{1}{3}\times\frac{50}{144}$

$= \frac{83}{149}$

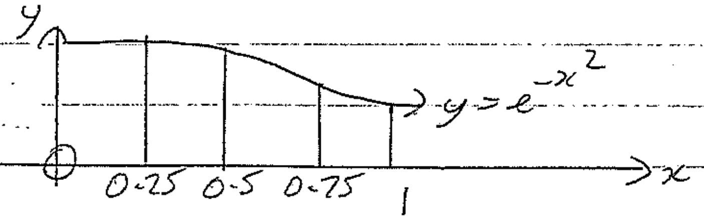
(iii) Winning tickets different colours

$= 1-(RR + GG + BB)$

$= 1-3\times\frac{1}{3}\times\frac{49}{149}$

$= \frac{100}{144}.$

Q.(13)(a) $\int_0^1 e^{-x^2}\,dx$ → 5 function values → 4 intervals of 0·25.




By $A \doteqdot \frac{h}{2}\left[y_0+y_4+2(y_1+y_2+y_3)\right]$

$= \frac{0{\cdot}25}{2}\left[e^{-0^2}+e^{-1^2}+2\left(e^{-(0{\cdot}25)^2}+e^{-(0{\cdot}5)^2}+e^{-(0{\cdot}75)^2}\right)\right]$

$= \frac{1}{8}\left[e^{0}+e^{-1}+2\left(e^{-\frac{1}{16}}+e^{-\frac{1}{4}}+e^{-\frac{9}{16}}\right)\right]$

$\doteqdot 0{\cdot}742984\ldots$

$= 0{\cdot}7430\ u^2$ (4DP)

(b) $A = \frac{h}{3}\left[y_0+y + 4(y_1+y_3)+2y_2\right]$

$\doteqdot \frac{80}{3}\left[85+0+4(92+110)+2\times107\right]$

$A \doteqdot 29\,520 m^2$

(c)(i) $y = 5xe^x$

$y' = u'v + v'u$

$= 5e^x + 5xe^x$

(ii) Hence $\int 5e^x+5xe^x\,dx = 5xe^x$ [+C].

$\int 5e^x\,dx + \int 5xe^x\,dx = 5xe^x$ [+C].

$5e^x + \int 5xe^x\,dx = 5xe^x$ [+C].

$\int 5xe^x\,dx = 5xe^x - 5e^x + C.$

Q.(13)(d) $V = \pi \int_{-1}^{2} (e^{3x})^2 \cdot dx$

$= \pi \int_{-1}^{2} e^{6x} \cdot dx$

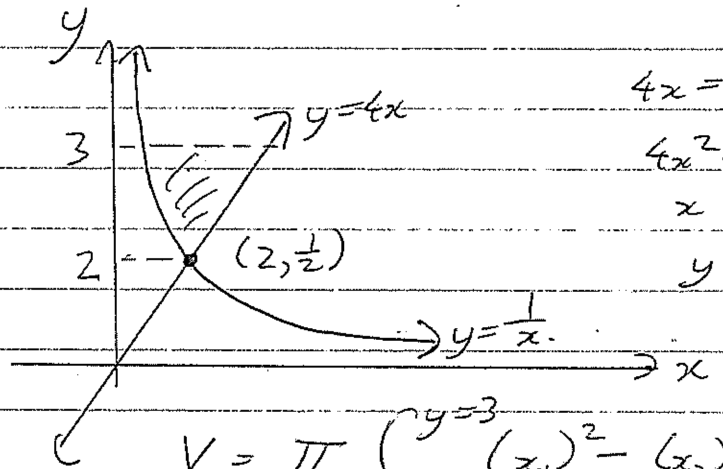
$= \pi \left[ \frac{1}{6} e^{6x} \right]_{-1}^{2}$

$= \frac{\pi}{6} \left[ e^{6\times 2} - e^{6\times -1} \right]$

$= \frac{\pi}{6} \left[ e^{12} - \frac{1}{e^6} \right]$

$\doteqdot 85\ 218\ u^3$ (nearest cubic unit).

(e)




$4x = \frac{1}{x}$

$4x^2 = 1$

$x = \pm\frac{1}{2}$

$y = \pm 2$

$V = \pi \int_{y=2}^{y=3} (x_1)^2 - (x_2)^2 \cdot dy$

$y = 4x \qquad y = \frac{1}{x}$

$\frac{1}{4}y = x \qquad \frac{1}{y} = x$

$\therefore \quad V = \pi \int_{y=2}^{y=3} (\tfrac{1}{4}y)^2 - (\tfrac{1}{y})^2 \cdot dy$

$= \pi \int_{2}^{3} \frac{1}{16}y^2 - \frac{1}{y^2} \cdot dy$

$= \pi \left[ \frac{y^3}{48} + \frac{1}{y} \right]_{2}^{3}$

$= \pi \left[ \frac{3^3}{48} + \frac{1}{3} - \frac{2^3}{48} - \frac{1}{2} \right]$

$= \frac{11\pi}{48}\ u^3$ or $= 0{\cdot}72u^3$ (2DP).

(14)(a)(i) 12TH month $T_n = a + (n-1)d$

$T_{12} = 40 + 11 \times 15$

$= \$205.$

(ii) After 1 year: $S_n = \frac{n}{2}(a+l)$

$= 6(40+205)$

$= \$1470.$

But he also has \$500 so total $= \$1970.$

(iii) $500 + \frac{n}{2}\left[2\times 40 + (n-1)\times 15\right] = 7000$

$\frac{n}{2}\left[65+15n\right] = 6500$ $\times\frac{2}{5}$ & expand.

$3n^2 + 13n - 2600 = 0$

$n = \frac{-13 \pm \sqrt{31369}}{2\times 3}$

$= 27.352..$

→ It will take 28 months.

(14)(b)(i) $T_n = ar^{n-1}$

$= 3000 \times 1{\cdot}07^5$

$= 4207{\cdot}655\ldots$

$\rightarrow$ 4208 people attended (nearest person)

(would take 4207 or 4210 (nearest 10 people))

(ii) GP: $S_8 = \dfrac{a(r^n - 1)}{r-1}$

$= \dfrac{3000(1{\cdot}07^8 - 1)}{0{\cdot}07}$

$= 30\ 779$ people (nearest person)

or $30\ 780$ people (nearest 10)

(c) $y = e^{-2x}$

$y' = -2e^{-2x}$

At $x = -1$, $y' = -2e^{-2\times -1}$

$= -2e^2$

Also at $x = -1$, $y = e^{-2\times -1} = e^2$

By $y - y_1 = m(x - x_1)$

$y - e^2 = -2e^2(x+1)$

$y - e^2 = -2e^2x - 2e^2$

$y = -2e^2x - e^2$

OR $2e^2x + y + e^2 = 0$

(d) $y = e^{2x} + 3e^{-x}$

$y' = 2e^{2x} - 3e^{-x}$

$y'' = 4e^{2x} + 3e^{-x}$

$\therefore \dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} - 2y$

$= (4e^{2x} + 3e^{-x}) - (2e^{2x} - 3e^{-x}) - 2(e^{2x} + 3e^{-x})$

$= 4e^{2x} + 3e^{-x} - 2e^{2x} + 3e^{-x} - 2e^{2x} - 6e^{-x}$

$= 0 = RHS.$

QED.

Q.(15)(a)(i) When $x=-1$

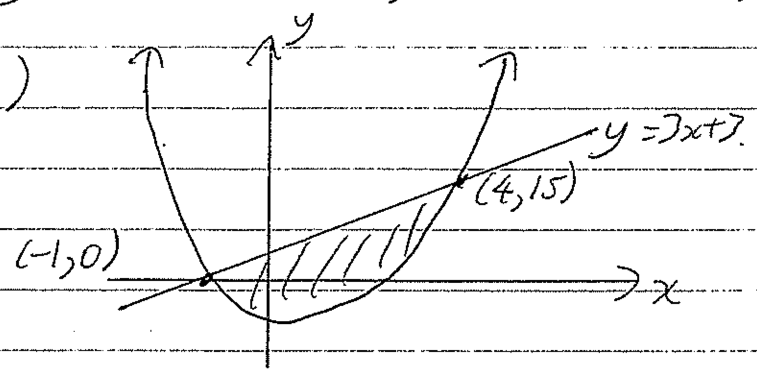
$y=x^2-1$ $\quad$ $y=3x+3$
$=(-1)^2-1$ $\quad$ $=3\times-1+3$
$=0$ $\quad$ $=0$.

When $x=4$

$y=x^2-1$ $\quad$ $y=3x+3$
$=4^2-1$ $\quad$ $=3\times4+3$
$=15$. $\quad$ $=15$.

They intersect at $(-1,0)$ & $(4,15)$.

(ii)




$$A=\int_{-1}^{4}(3x+3)-(x^2-1).dx$$

$$=\int_{-1}^{4}3x-x^2+4.dx$$

$$=\left[\frac{3}{2}x^2-\frac{1}{3}x^3+4x\right]_{-1}^{4}$$

$$=\left[\frac{3}{2}\times4^2-\frac{1}{3}\times4^3+4\times4\right]-\left[\frac{3}{2}\times(-1)^2-\frac{1}{3}\times(-1)^3+4\times-1\right]$$

$$=\frac{56}{3}\quad--\frac{13}{6}$$

$$=\frac{125}{6}u^2 \text{ or } 20\tfrac{5}{6}u^2.$$

(b)(i) $y=(x+1)e^x$.

y intercept $[x=0]$.
$=(0+1)\times e^0$
$=1$.

x intercept $[y=0]$
$(x+1)e^x=0$
As $e^x\neq0$ for all real $x$,
$x+1=0$
$x=-1$.

Q.(15)(b)(ii) Turning Points:

$$y' = u'v + v'u$$
$$= 1\times e^x + e^x(x+1)$$
$$= (x+2)e^x.$$

$y'=0$: $(x+2)e^x = 0$

$x+2 = 0 \Rightarrow x = -2.$

At $x=-2$, $y = (x+1)e^x$
$$= (-2+1)e^{-2}$$
$$= -\frac{1}{e^2}.$$

Nature of turning points

$$y'' = u'v + v'u$$
$$= 1\times e^x + e^x\times(x+2)$$
$$= (x+3)e^x.$$

At $x=-2$, $y'' = (-2+3)\times e^{-2}$
$$= \frac{1}{e^2} > 0$$

$\therefore$ MINIMUM turning point at $\left(-2, -\frac{1}{e^2}\right)$

(iii) Points of inflexion:

$$y'' = 0$$
$$(x+3)e^x = 0$$
$$x+3 = 0 \Rightarrow x = -3.$$

At $x=-3$, $y = (x+1)e^x$
$$= (-3+1)e^{-3}$$
$$= -\frac{2}{e^3}.$$

Testing concavity to LEFT of $x=-3$.

At $x=-4$, $y'' = (-4+3)e^{-4}$
$$= -\frac{1}{e^4} < 0$$

$\rightarrow$ Concave DOWN to left.

Already tested concavity to RIGHT $\Rightarrow$ at $x=-2$,

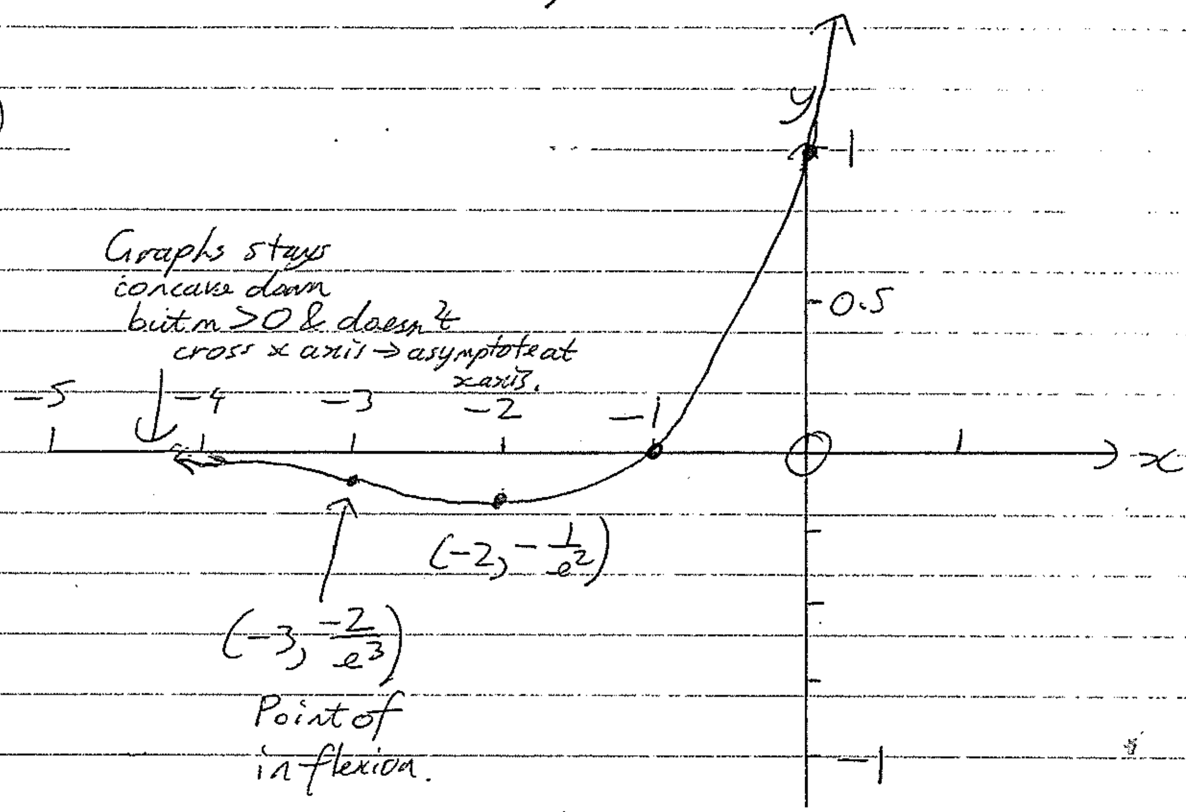
$y'' = \frac{1}{e^2} > 0.$

Concavity CHANGES

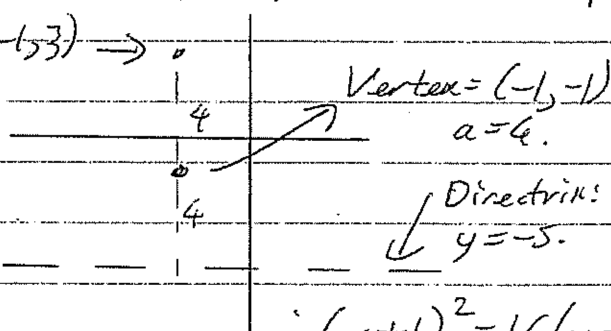
PTO $\rightarrow$

Q (15)(b)(iii) [cont] $\therefore$ Point of inflexion at $\left(-3, -\frac{2}{e^3}\right)$

(iv)




Q (16)(a) → Equidistant from point & line = parabola




$\therefore (x+1)^2 = 16(y+1)$.

(b)(i) $A = (x, x^2+2x+5)$ $B = (x, 4x-x^2)$

Distance $AB = (x^2+2x+5) - (4x-x^2)$

$= 2x^2 - 2x + 5$.

PTO →

Q.(16)(b)(ii) Minimum AB:

$$\frac{d}{dx}(AB) = 4x-2.$$

Finding minimum: $\frac{d}{dx}(AB) = 0$

$$4x-2=0$$
$$x=\frac{1}{2}.$$

Testing to see if minimum:

$$\frac{d^2}{dx^2}(AB) = 4 > 0$$

$\therefore$ Minimum AB at $x=\frac{1}{2}$.

At $x=\frac{1}{2}$, $AB = 2x^2-2x+$

$$= 2\times(\tfrac{1}{2})^2-2\times\tfrac{1}{2}+5$$
$$= \frac{9}{2} \text{ or } 4\tfrac{1}{2}.$$

$\therefore$ Minimum distance $AB = 4\frac{1}{2}$ units.

(c)(i) $1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots$

$a=1$, $r=\frac{1}{3}$.

By $S_n = \dfrac{a(1-r^n)}{1-r}$

$$S_n = \frac{1(1-(\frac{1}{3})^n)}{1-\frac{1}{3}}$$

$$S_n = \frac{3}{2}\left[1-(\tfrac{1}{3})^n\right].$$

PTO →

Q(6)(c)(ii)(A)

$$1 + \frac{2}{3} + \frac{3}{9} + \frac{4}{27} + \frac{5}{81}$$

$$= 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} \quad (1)$$

$$+ \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} \quad (2)$$

$$+ \frac{1}{9} + \frac{1}{27} + \frac{1}{81} \quad (3)$$

$$+ \frac{1}{27} + \frac{1}{81} \quad (4)$$

$$+ \frac{1}{81}. \quad (5)$$

Using $S_n = \frac{a(1-r^n)}{1-r}$

(1) $= \frac{1[1-(\frac{1}{3})^5]}{1-\frac{1}{3}} = \frac{3}{2} - \frac{3}{2} \times (\frac{1}{3})^5 = \frac{3}{2} - \frac{1}{2} \times (\frac{1}{3})^4$

(2) $= \frac{\frac{1}{3}[1-(\frac{1}{3})^4]}{1-\frac{1}{3}} = \frac{1}{2} - \frac{1}{2} \times (\frac{1}{3})^4 = \frac{1}{2} - \frac{1}{2} \times (\frac{1}{3})^4$

(3) $= \frac{\frac{1}{9}[1-(\frac{1}{3})^3]}{1-\frac{1}{3}} = \frac{1}{6} - \frac{1}{6} \times (\frac{1}{3})^3 = \frac{1}{6} - \frac{1}{2} \times (\frac{1}{3})^4$

(4) $= \frac{\frac{1}{27}[1-(\frac{1}{3})^2]}{1-\frac{1}{3}} = \frac{1}{18} - \frac{1}{2} \times (\frac{1}{3})^4$

(5) $= \frac{\frac{1}{81}[1-(\frac{1}{3})]}{1-\frac{1}{3}} = \frac{1}{54} - \frac{1}{2} \times (\frac{1}{3})^4$

(1) + (2) + (3) + (4) + (5)

$$= \left(\frac{3}{2} + \frac{1}{2} + \frac{1}{6} + \frac{1}{18} + \frac{1}{54}\right) - \frac{5}{2} \times \left(\frac{1}{3}\right)^4$$

By $S_n = \frac{a(1-r^n)}{1-r^n}$

$$= \frac{\frac{3}{2}[1-(\frac{1}{3})^5]}{1-\frac{1}{3}} - \frac{5}{2} \times \left(\frac{1}{3}\right)^4$$

$$= \frac{9}{4}\left[1 - \left(\frac{1}{3}\right)^5\right] - \frac{5}{2} \times \left(\frac{1}{3}\right)^4$$

= RHS QED.

(16)(c)(ii)(B) $S_5 = \frac{9}{4}\left[1-\left(\frac{1}{3}\right)^5\right] - \frac{5}{2}\times\left(\frac{1}{3}\right)^4$.

$$S_n = \frac{9}{4}\left[1-\left(\frac{1}{3}\right)^n\right] - \frac{n}{2}\times\left(\frac{1}{3}\right)^{n-1}.$$

(C) Limiting Sum: As $n \to \infty$, $\left(\frac{1}{3}\right)^n$ & $\left(\frac{1}{3}\right)^{n-1} \to 0$

$\therefore$ Limiting Sum $= \frac{9}{4}\times 1$

$= \frac{9}{4}$.

Here endeth the solutions!!!